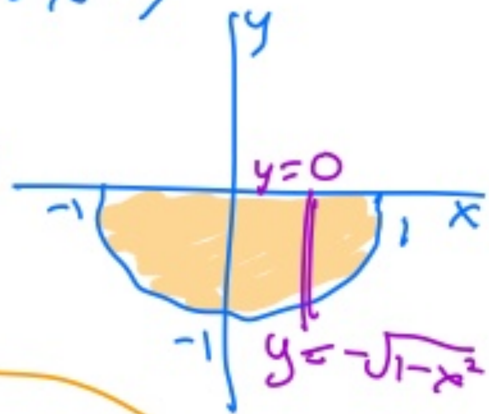
Hold on to homework! :)

Example

Find $\displaystyle\iint_R e^{x^2+y^2}\,dx\,dy$

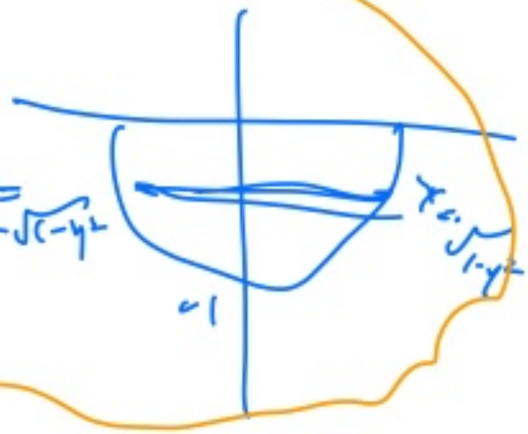
where R is this region



Rectangular

$$\int_{x=-1}^{1}\int_{y=-\sqrt{1-x^2}}^{0} e^{x^2+y^2}\,dy\,dx$$

or $$\int_{y=-1}^{0}\int_{x=-\sqrt{1-y^2}}^{\sqrt{1-y^2}} e^{x^2+y^2}\,dx\,dy$$



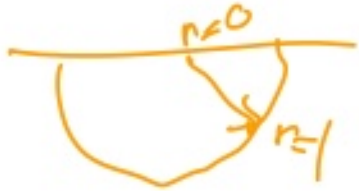
Polar

$$f(x,y)=e^{x^2+y^2}=e^{r^2}$$

$$dx\,dy=r\,dr\,d\theta$$

$$0\le r\le 1$$

$$-\pi\le\theta\le 0$$



$$\int_{\theta=-\pi}^{0}\int_{r=0}^{1} \underline{e^{r^2}\,r\,dr}\,d\theta$$

$u=r^2$ $\quad du=2r\,dr$

$r\,dr=\frac{1}{2}du$

$0\le u\le 1$

$$\int_{\theta=-\pi}^{0}\left(\int_{u=0}^{1}\frac{1}{2}e^{u}\,du\right)d\theta$$

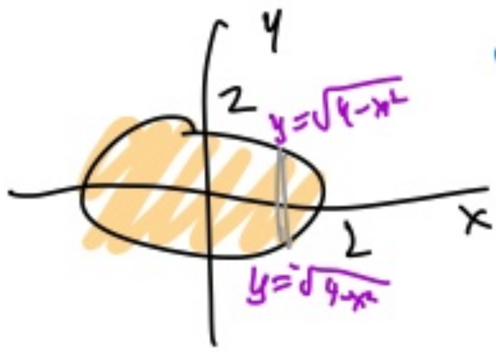
$$\int_{\theta=\pi}^{0}\left(\frac{1}{2}e^{u}\Big|_{0}^{1}\right)d\theta$$

$$=\int_{-\pi}^{0}\left(\frac{e}{2}-\frac{1}{2}\right)d\theta$$

$$\left(\frac{e}{2}-\frac{1}{2}\right)\theta\Big|_{-\pi}^{0}$$

$$=\boxed{\frac{(e-1)\pi}{2}}$$

3.4 $S=\{(x,y): x^2+y^2\leq 4 \quad 0\leq z\leq 4x+8\}$



upper
$z=4x+8$
lower
$z=0$

$$\text{Volume}=\iiint_S 1\,dx\,dy\,dz$$

$$\text{Volume} = \int_{x=-2}^{2}\int_{y=-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{z=0}^{z=4x+8} 1 \; dz\, dy\, dx$$

$$z\Big|_0^{4x+8}$$

$$= \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4x+8)\, dy\, dx$$

$$= \int_{-2}^{2} (4x+8)\, y\Big|_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$$

$$= \int_{-2}^{2} 2(4x+8)\sqrt{4-x^2}\; dx$$

...

Polar:

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

$f(x,y,z) =$

$(4r\cos\theta + 8)$

$dx\,dy = r\,dr\,d\theta$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{2} (4r\cos\theta + 8)\, r\, dr\, d\theta$$

$4r^2\cos\theta + 8r$

$$\int_{\theta=0}^{2\pi} \left(\frac{4r^3}{3}\cos\theta + 4r^2\right)\Big|_0^2 d\theta$$

$$= \int_{\theta=0}^{2\pi} \left(\frac{32}{3}\cos\theta + 16\right) d\theta$$

$$= 16(2\pi) = \boxed{32\pi}.$$